# Band Electronic Structure of the Purple Potassium Molybdenum Bronze $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ 

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#### Abstract

The electronic properties of potassium molybdenum purple bronze $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ were examined by performing tight-binding band calculations on a single $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer. Our calculations show the presence of three partially filled d-block bands, which are essentially derived from the $\mathrm{t}_{2 \mathrm{~g}}$ orbitals of the $\mathrm{MoO}_{6}$ octahedra belonging to the innermost two sublayers of $\mathrm{Mo}_{6} \mathrm{O}_{17}$. The Fermi surfaces of the three partially filled bands are all closed, and thus $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ is a two-dimensional metal. The charge density wave of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$, which sets in at 120 K , is caused by the nesting of one of the three Fermi surfaces, and the remaining two provide electron and hole carriers below 120 K . The $\mathrm{t}_{2 \mathrm{~g}}$-block bands of the innermost two sublayers of $\mathrm{Mo}_{6} \mathrm{O}_{17}$ were examined by analyzing the band orbitals at a few high-symmetry wave vector points. The dispersion characteristics of those bands are determined by whether or not the orbitals of bridging oxygen atoms can mix with the molybdenum $\mathrm{t}_{2 \mathrm{~g}}$ orbitals.


Molybdenum bronzes are a class of solid oxide phases with a range of composition $\mathrm{A}_{x} \mathrm{Mo}_{y} \mathrm{O}_{z}$, where A is an alkali metal or $\mathrm{Tl},{ }^{2}$ and exhibit intense color. There are three well-defined molybdenum bronzes: The red bronze $\mathrm{A}_{0.33} \mathrm{MoO}_{3}(\mathrm{~A}=\mathrm{K})^{3.4}$ and the blue bronze $\mathrm{A}_{0.3} \mathrm{MoO}_{3}(\mathrm{~A}=\mathrm{K}, \mathrm{Rb}, \mathrm{Tl})^{3,5,6}$ contain separated metal-oxygen layers of composition $\mathrm{MoO}_{3}$ that are constructed from $\mathrm{MoO}_{6}$ octahedra. The potassium purple bronze $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}{ }^{7-9}$ has separated metal-oxygen layers of composition $\mathrm{Mo}_{6} \mathrm{O}_{17}$, which are made up of both $\mathrm{MoO}_{6}$ octahedra and $\mathrm{MoO}_{4}$ tetrahedra. The sodium purple bronze $\mathrm{Na}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}{ }^{10}$ and the thallium purple bronze $\mathrm{TlMo}_{6} \mathrm{O}_{17}{ }^{11}$ are similar in structure to the potassium purple bronze, but the lithium purple bronze $\mathrm{Li}_{0.9}{ }^{-}$
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$\mathrm{Mo}_{6} \mathrm{O}_{17}{ }^{12}$ differs from the other purple bronzes in that it has no separated metal-oxygen layers. ${ }^{12 a}$

An alkali metal atom of a bronze $\mathrm{A}_{x} \mathrm{Mo}_{y} \mathrm{O}_{z}$ donates its valence electron to the d-block bands of the transition metal Mo, so the nature of these bands determines whether the bronze is a metal or an insulator. The red bronze $\mathrm{A}_{0.33} \mathrm{MoO}_{3}$ is a semiconductor at all temperatures, ${ }^{4 \mathrm{~b}}$ while the blue bronze $\mathrm{A}_{0.3} \mathrm{MoO}_{3}$ is a qua-si-one-dimensional (1D) metal above 180 K , below which it becomes a semiconductor due to a charge density wave (CDW) formation. ${ }^{6}$ This difference in the electrical properties of the two bronzes originates from the fact that their $\mathrm{MoO}_{3}$ layers do not have identical structures.

The potassium purple bronze $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ is a two-dimensional (2D) metal above 120 K , at which it undergoes a CDW phase transition. ${ }^{8,9}$ In contrast to the case of the blue bronze, however, the potassium purple bronze remains a 2D metal after the CDW phase transition. ${ }^{8}$ Diffuse X-ray scattering and electron diffraction studies show the occurrence of superlattice spots at $q_{a}=\left(a^{*} / 2\right.$, $0,0), q_{b}=\left(0, b^{*} / 2,0\right)$, and $q_{a+b}=\left(a^{*} / 2, b^{*} / 2,0\right)$ below 120 K. ${ }^{9 b}$ That is, the CDW formation increases the unit cell dimension fourfold, from $(a, b, c)$ to $(2 a, 2 b, c)$. The CDW vectors $q_{a}, q_{b}$, and $q_{a+b}$ are commensurate within $4 \%$ experimental accuracy. ${ }^{\circ b}$ According to the thermoelectric power measurements, the major carriers for electrical transport in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ are electrons and holes above and below 120 K , respectively, and the CDW removes not all but about $50 \%$ of the carrier concentration. ${ }^{8 a}$ Hall effect measurements show the presence of both electrons and holes below 120 K . According to these observations, several partially filled d-block bands are present in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$. In electrical, CDW, and other physical properties, $\mathrm{Na}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}{ }^{10}$ and $\mathrm{TlMo}_{6} \mathrm{O}_{17}{ }^{11}$ are similar to $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$.

In understanding the properties of the potassium, sodium, and thallium molybdenum purple bronzes, it is crucial to know their band electronic structures. In the present study, we report our tight-binding band calculations on $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ based upon the extended-Hückel method. ${ }^{13}$ The results of our band calculations on $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ should also be valid for $\mathrm{Na}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ and Tl $\mathrm{Mo}_{6} \mathrm{O}_{17}$. It is noted that similar band calculations performed on the blue bronze ${ }^{6 f}$ led to results in excellent agreement with experiment. The atomic parameters employed in the present work

[^0]were taken from ref 6 f. In what follows, we first examine the structural characteristics of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ and then discuss its dblock band electronic structure and its associated Fermi surfaces. Finally, we analyze the essential features of the calculated d-block bands from the viewpoint of orbital interactions.

## Crystal Structure

It is convenient to describe the crystal structure of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ in terms of the "building unit" $\mathrm{Mo}_{4} \mathrm{O}_{21}$ (1),


1
which is constructed from four $\mathrm{MoO}_{6}$ octahedra by sharing the axial oxygen atoms. Shown in $\mathbf{2 a}$ is a schematic representation of the hexagonal layer $\mathrm{Mo}_{4} \mathrm{O}_{15}\left(a=b \neq c, \alpha=\beta=90^{\circ}, \gamma=\right.$ $120^{\circ}$ ) constructed from the $\mathrm{Mo}_{4} \mathrm{O}_{21}$ chains: Along the crys-

tallographic $b$ axis, every adjacent pair of $\mathrm{Mo}_{4} \mathrm{O}_{21}$ chains share the equatorial oxygen atoms of three $\mathrm{MoO}_{6}$ octahedra. Though not shown explicity, every $\mathrm{MoO}_{6}$ octahedron of 2a forms a zigzag chain $\mathrm{Mo}_{2} \mathrm{O}_{10}$ (3), along the crystallographic $a$ axis. The $\mathrm{Mo}_{4} \mathrm{O}_{15}$

layer $\mathbf{2 a}$ can alternatively be represented by $\mathbf{2 b}$, which makes it clear that the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer $\mathbf{2}$ consists of four sublayers of $\mathrm{MoO}_{6}$ octahedra parallel to the $a b$ plane. When the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer 2 is viewed along the crystallographic $c$ axis (i.e., perpendicular to the layer as in the case of, for example, 6 discussed later), every three nearest-neighbor $\mathrm{MoO}_{6}$ octahedra of its outer two sublayers form a triangle of oxygen atoms that can be used as a face of an $\mathrm{MoO}_{4}$ tetrahedron. When all such oxygen triangles of the outer sublayers of the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer are capped by $\mathrm{MoO}_{4}$ tetrahedra, one obtains the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer found in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$. It is between these $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layers where the potassium cations $\mathrm{K}^{+}$reside.

Thus there exist three types of Mo atoms in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ : the Mo atoms of the $\mathrm{MoO}_{4}$ tetrahedra $\left(\mathrm{Mo}_{1}\right)$, those of the $\mathrm{MoO}_{6}$ octahedra in the outer two sublayers of the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer $\left(\mathrm{Mo}_{11}\right)$, and those of the $\mathrm{MoO}_{6}$ octahedra in the inner two sublayers of the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer $\left(\mathrm{Mo}_{\mathrm{III}}\right)$. In connection with the electrical and other physical properties of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$, it is essential to know how the delectrons are distributed among the three different Mo atoms. Empirically, this question is answered by performing a Zachariasen analysis ${ }^{14}$ of metal-oxygen bond lengths, in which the difference between the length $r_{i}$ of a given bond $i$ and its
standard bond length $r_{0}$ is related to the strength $s_{i}$ of that bond as follows:

$$
\begin{equation*}
r_{i}-r_{0}=\alpha \ln s_{i} \tag{1}
\end{equation*}
$$

where $\alpha$ is a positive constant and the $s_{i}$ value means the number of electrons associated with bond $i$. Thus the sum of the $s_{i}$ values for all the bonds $i$ around each metal is equal to the total number of $d$ electrons the metal lost in forming those bonds (i.e., the formal oxidation state of the metal). When the constants $r_{0}$ and $\alpha$ for $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ are chosen such that the oxidation state of $\mathrm{Mo}_{1}$ becomes +6 , the oxidation states of $\mathrm{Mo}_{\mathrm{II}}$ and $\mathrm{Mo}_{\text {III }}$ are calculated to be +5.8 and +5.1 , respectively. ${ }^{7}$ In other words, the $d$ electrons of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ reside primarily in the $\mathrm{MoO}_{6}$ octahedra of $\mathrm{Mo}_{111}$. Therefore, the inner two sublayers of the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer are expected to play an important role in determining the electronic structure of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer and hence that of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$.
The inner two sublayers of $\mathbf{2}$ are schematically shown in 4, which has the composition of $\mathrm{Mo}_{2} \mathrm{O}_{9}$. This layer may be considered to originate from the "building unit" $\mathrm{Mo}_{2} \mathrm{O}_{11}, \mathbf{5 a}$. An



5a


5 c

5b
alternative view of $\mathbf{5 a}$ is shown in $\mathbf{5 b}$. A projection view of $\mathbf{5 b}$ along the direction perpendicular to a triangular face of the $\mathrm{MoO}_{6}$ octahedra is given by 5 c . With this representation of the $\mathrm{Mo}_{2} \mathrm{O}_{11}$ unit, a projection view of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer $\mathbf{4}$ along the $c$ axis is given in 6. It is important to recognize that the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer


6 is constructed from the $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chains 3 upon sharing the axial oxygen atoms as illustrated in 7. Due to symmetry, there are three different ways of making $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chains in 6: The $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chains are aligned along the $a-, b$-, and $(a+b)$-axis directions in $7 \mathrm{a}, 7 \mathrm{~b}$, and 7 c , respectively.



## Band Electronic Structure

For a 2D hexagonal lattice (i.e., $a=b, \gamma=120^{\circ}$ ) such as the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer 6, the $\mathrm{Mo}_{4} \mathrm{O}_{15}$ layer 2, or the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer, the first Brillouin zone of its reciprocal lattice is a regular hexagon centered at the origin $\Gamma$ as shown in Figure 1. ${ }^{15}$ In units of the reciprocal vectors $a^{*}$ and $b^{*}$, the points $\Gamma, \mathrm{M}, \mathrm{K}$, and $\mathrm{K}^{\prime}$ are defined as follows: $\Gamma=(0,0), \mathrm{M}=\left(a^{*} / 2,0\right), \mathrm{K}=\left(a^{*} / 3, b^{*} / 3\right)$, and $\mathrm{K}^{\prime}$ $=\left(a^{*} / 2, b^{*} / 2\right)$. In this section, we examine the d-block bands of the "ideal" and/or the "real" structures of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer and the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer calculated for wave vectors $k$ along the $\Gamma \rightarrow$ $\mathrm{M} \rightarrow \mathrm{K} \rightarrow \Gamma$ line. In our discussion, an "ideal" layer refers to one in which $\mathrm{MoO}_{6}$ octahedra or $\mathrm{MoO}_{4}$ tetrahedra are regular in shape, while a "real" layer refers to one in which $\mathrm{MoO}_{6}$ octahedra or $\mathrm{MoO}_{4}$ tetrahedra have shapes as found in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$.

Just as the $\mathrm{t}_{2 \mathrm{~g}}$-block levels of an $\mathrm{MoO}_{6}$ octahedron lie below the $\mathrm{e}_{\mathrm{g}}$-block levels, the $\mathrm{t}_{2 \mathrm{~g}}$-block bands of the $\mathrm{MO}_{2} \mathrm{O}_{9}$ layer lie below the $\mathrm{e}_{\mathrm{g}}$-block bands. Figure 2 shows the $\mathrm{t}_{2 \mathrm{~g}}$-block bands calculated for the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer 6, where the two bands indicated by asterisks are each doubly degenerate. Thus, the $\mathrm{t}_{2 \mathrm{~g}}$-block bands consist of six bands, as expected from the presence of two metal atoms in the unit cell. The $\mathrm{t}_{2 \mathrm{~g}}$-block bands of the real $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer are shown in Figure 3, which are very close in character to those of the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer. The $\mathrm{MoO}_{6}$ octahedra of the real $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer are slightly distorted from a regular $\mathrm{MoO}_{6}$ octahedron, so the degeneracy in the $\mathrm{t}_{2 \mathrm{~g}}$-block bands of the ideal layer is somewhat lifted by the structural change to the real layer.

Shown in Figure 4 is the bottom portion of the d-block bands calculated for the real $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer. It is observed that the bottom three d-block bands of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer are essentially identical with those of the real $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer. The bottom three d-block bands of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer have the d-orbital character of primarily $\mathrm{Mo}_{111}$ atoms. The d-block bands of mainly $\mathrm{Mo}_{\mathrm{I}}$ and $\mathrm{Mo}_{\text {II }}$ character all lie above the bottom three d-block bands. With 2.9 d electrons per $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ (i.e., $\mathrm{K}^{+}{ }_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}{ }^{0.9-}$ ) or 2.9 d electrons to fill the d-block bands of Figure 4, only the bottom three bands are partially filled. This result is in agreement with the prediction of the Zachariasen analysis that the delectrons of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ reside primarily in the $\mathrm{Mo}_{\text {III }}$ atoms of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer. ${ }^{7}$
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Figure 1. First Brillouin zone of a 2D hexagonal lattice.


Figure 2. $\mathrm{t}_{2 \mathrm{~g}}$ block bands of the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer.


Figure 3. $\mathrm{t}_{2 \mathrm{~g}}$ block bands of the real $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer.


Figure 4. Bottom portion of the d-block bands of the real $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer.
For a partially filled band, a certain region of wave vectors of the Brillouin zone leads to occupied band levels, and another region to unoccupied band levels. The Fermi surface of a partially filled band is the boundary surface of wave vectors that separate the occupied wave vector region from the unoccupied wave vector region. When the Fermi level is close to the top (bottom) of a partially filled band, the unoccupied (occupied) region of wave vectors is referred to as a hole (electron) pocket. If a piece of a Fermi surface is related to another piece by a mere translation of wave vector $q$, the two pieces are said to be nested by $q$. Such a nesting is favorable for a CDW of wave vector $q$.

The Fermi surfaces calculated for the three partially filled bands of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer are shown in Figure 5, where the shaded and unshaded regions signify the occupied and unoccupied regions, respectively. As shown in parts a and c of Figure 5, the first and the third d-block bands (from the bottom) of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer give hole pockets around K and an electron pocket around $\Gamma$, respectively. The Fermi surfaces of these two bands do not have good nesting. The second d-block band (from the bottom) of the


Figure 5. Fermi surfaces for the first, second, and third d-block bands (from the bottom) of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer in (a), (b), and (c), respectively.


Figure 6. Two pairs of nested pieces of the Fermi surface associated with the second d-block band of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer.
$\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer leads to the Fermi surface in Figure 5 b , which gives rise to six pairs of nested pieces as illustrated for two of them in Figure 6. In our calculations, the $a^{*}$ component of the nesting vectors $q_{1}$ and $q_{2}$ is very close to $a^{*} / 2$. The vectors $q_{1}$ and $q_{2}$ cancel out their $b^{*}$ components and therefore are responsible for the superlattice spot at $q_{a}=\left(a^{*} / 2,0,0\right)$ in $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$. In a similar manner, the remaining four nesting vectors account for the superlattice spots at $q_{b}=\left(0, b^{*} / 2,0\right)$ and $q_{a+b}=\left(a^{*} / 2, b^{*} / 2,0\right)$.

All three Fermi surfaces of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer are closed, so that the potassium purple bronze is a 2 D metal. The CDW of $\mathrm{K}_{0.9^{-}}$ $\mathrm{Mo}_{6} \mathrm{O}_{17}$ that sets in at 120 K is caused by the nesting of the Fermi surface resulting from the second d-block band. This CDW does not affect the Fermi surfaces associated with the first and third d-block bands. Consequently, $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ remains a 2 D metal below 120 K , and its carriers below 120 K consist of electrons and holes as expected from Figure 5a,c, respectively. All of these results are in good agreement with experiment. ${ }^{8,9}$

## $\mathrm{t}_{\mathbf{2 g}}$-Block Bands of the Ideal $\mathbf{M o}_{\mathbf{2}} \mathrm{O}_{\mathbf{9}}$ Layer

As discussed in the previous section, the bottom three d-block bands of the $\mathrm{MO}_{6} \mathrm{O}_{17}$ layer are responsible for the 2D metallic, $C D W$, and other physical properties of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$. In understanding these properties, it is crucial to examine how those bands come about. In this section, we probe this question by analyzing the nature of the $t_{2 g}$-block bands of the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer since the bottom portion of these bands is essentially similar to the three partially filled bands of the $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer and since the high symmetry of the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer makes our analysis simple.
A. Metal-Ligand Interactions. The three $\mathrm{t}_{2 \mathrm{~g}}$ levels of an $\mathrm{MoO}_{6}$ octahedron are shown in 8 , where small antibonding contributions from the ligand oxygen orbitals are omitted for simplicity. For


8
our discussion, it is crucial to recall that in d-block levels of most transition-metal compounds, the ligand orbitals combine out-
of-phase (i.e., in an antibonding way) with the metal d-orbitals. Therefore, the energy of a d-block level is raised or lowered when the ligand orbital contribution increases or decreases, respectively.

In the $\mathrm{MO}_{2} \mathrm{O}_{9}$ layer, the Mo atoms interact with one another via $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges. As shown in 7, $\mathrm{Mo}_{2} \mathrm{O}_{9}$ can be regarded as being derived from the $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chains 3 upon sharing their axial oxygen atoms. Thus there are two types of $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges in the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer: One is within each $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chain, and the other is between adjacent $\mathrm{Mo}_{2} \mathrm{O}_{10}$ chains. Along the intra- and interchain $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges, the p orbitals of the bridging oxygen may interact with the adjacent metal d orbitals as depicted in 9 and 10 , respectively. In $9 \mathrm{a}, 10 \mathrm{a}$, or 10 b , all the interacting orbitals are contained in a plane, while this is not the case in $\mathbf{9 b}$ or 9 c . Hence, the $\pi$-type overlap present in $\mathbf{9 a}, \mathbf{1 0 a}$, or $\mathbf{1 0 b}$ is greater than that in $\mathbf{9 b}$ or $9 \mathbf{9}$.

$9 a$


9b



10a


10b


10c

For the bridging oxygen p orbitals to mix with the adjacent metal d orbitals in 9 and $\mathbf{1 0}$, the two metal orbitals of an MoO -Mo bridge must have an in-phase combination (e.g., see the orbital $\phi_{1}{ }^{+}$in 12). Otherwise, the bridging oxygen p orbital cannot mix with the metal d orbitals (e.g., see $\phi_{1}^{-}$in 12). ${ }^{17}$ In 10 c , the metal orbitals have a $\delta$-type interaction, and thus none of the bridging oxygen orbitals can mix with the metal d orbitals (e.g., see 15).
B. Band Dispersion. Point $\mathrm{K}^{\prime}$ in Figure 1 is equivalent to $\mathrm{M}^{\prime}$ $=\left(-a^{*} / 2, b^{*} / 2\right)$, since $\mathrm{M}^{\prime}$ differs from $\mathrm{K}^{\prime}$ by an integral multiple of the reciprocal vector; ${ }^{15}$ i.e., $\mathrm{M}^{\prime}=\mathrm{K}^{\prime}-\left(a^{*}, 0\right)$. By symmetry, $M^{\prime}$ is equivalent to $M$, so that the $K \rightarrow M$ line of the Brillouin is equivalent to the $\mathrm{K} \rightarrow \mathrm{K}^{\prime}$ line. Thus, in our discussion, the $\Gamma$ $\rightarrow \mathrm{K} \rightarrow \mathrm{M}$ line can be replaced by the $\Gamma \rightarrow \mathrm{K}^{\prime}$ line. Then essential features of the d-block bands of the ideal $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer in Figure 2 are summarized as follows: (a) The nondegenerate bands a and b are flat along $\Gamma \rightarrow \mathrm{M}$ but strongly dispersive along $\Gamma \rightarrow \mathrm{K}^{\prime}$. In addition, the two bands cross each other along $\Gamma \rightarrow \mathrm{K}^{\prime}$. (b) The doubly degenerate bands c and d are both equally dispersive along $\Gamma \rightarrow M$ and along $\Gamma \rightarrow \mathrm{K}^{\prime}$, the extent of the dispersion being half that of the bands a and b along $\Gamma \rightarrow \mathrm{K}^{\prime}$. In the following we examine these trends in terms of orbital interaction analysis. ${ }^{17}$

1. Bands a and $b$. On going from $\Gamma$ to $M$, the phases between nearest-neighbor unit cell orbitals remain in-phase in one direction (e.g., $b$ axis in 7b) but change from in-phase to out-of-phase in the other direction (e.g., $a$ axis in $\mathbf{7 a}$ ). ${ }^{17}$ To simplify our representation of band orbitals, the "building unit" $\mathrm{Mo}_{2} \mathrm{O}_{11}$, 11, of the $\mathrm{Mo}_{2} \mathrm{O}_{12}$ chain 3 may be represented by $\mathbf{1 2}$. On the basis of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer 7 b , the bands a and b at $\Gamma=(0,0)$ are given by the orbitals 13a and 13b, respectively. And the bands a and bat M $=\left(a^{*} / 2,0\right)$ are given by the orbitals 14 a and 14 b , respectively. Note that 13a and 14 a are derived from the unit cell orbitals $\phi_{1}^{-}$ of 12 , while 13 b and $\mathbf{1 4 b}$ are derived from the unit cell orbitals

[^1]
11

12


13a


14a


13b

$14 b$
$\phi_{1}{ }^{+}$of 12. As summarized in Table I, the oxygen orbitals of the interchain $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges do not contribute to 13 and 14. Those of the intrachain Mo-O-Mo bridges do not contribute to 13 a and 14 a , but they do to 13 b and 14 b , thereby raising the energies of $\mathbf{1 3 b}$ and $\mathbf{1 4 b}$ with respect to those of 13a and 14a. Since the chain orbitals do not overlap significantly across the interchain $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges, 13a and $\mathbf{1 4 a}$ are nearly the same in energy, and so are 13b and 14b.

On going from $\Gamma$ to $\mathrm{K}^{\prime}$, the phases between nearest-neighbor unit cell orbitals change from in-phase to out-of-phase in both the $a$ - and $b$-axis directions. ${ }^{17}$ The $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer 7 c has the unit cell orbitals $\phi_{2}{ }^{+}$and $\phi_{2}{ }^{-}$shown in 15. The band orbitals resulting


$\phi_{2}^{+}$

$\phi_{2}^{-}$
15
from $\phi_{2}{ }^{+}$and $\phi_{2}{ }^{-}$at $\Gamma$ are given by $16 a$ and $16 b$, respectively, and those at $\mathrm{K}^{\prime}$ by $\mathbf{1 7 a}$ and $\mathbf{1 7 b}$, respectively. As can be seen from the p-orbital contributions of the bridging oxygens, summarized in Table I, 16a is lower in energy than 16b, but 17a is higher in energy than $\mathbf{1 7 b}$. With respect to the twofold rotation around

$16 a$


16b


17b
the ( $a+b$ ) axis of $\mathbf{7 c}, \mathbf{1 6 a}$ and $\mathbf{1 7 a}$ are symmetric, but $\mathbf{1 6 b}$ and 17b are antisymmetric. Therefore, on going from $\Gamma$ to $K^{\prime}, 16 a$ and 16 b are correlated to 17 a and 17 b , respectively, and thus the two correlation curves (i.e., the bands a and b) cross each other. This explains why the bands $a$ and $b$ are highly dispersive along $\Gamma \rightarrow K^{\prime}$.

Given the translational symmetry of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer, $\mathbf{1 6} \mathbf{a}$ and 16b are identical with 13a and 13b, respectively, while 17a and $\mathbf{1 7 b}$ are identical with $\mathbf{1 4 b}$ and 14 a , respectively. The latter observation is particularly interesting in that the band orbitals at M (derived from the layer representation $\mathbf{7 b}$ ) are identical with those at $K^{\prime}$ (derived from the layer representation 7c). From the viewpoint of band orbitals, this is why the M and $\mathrm{K}^{\prime}$ points are equivalent for a hexagonal lattice.
2. Bands $\mathbf{c}$ and d . On the basis of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ layer $\mathbf{7 b}$, we now examine the degenerate bands c and d along $\Gamma \rightarrow \mathrm{M}$, which are derived from the unit cell orbitals $\phi_{3}{ }^{+}$and $\phi_{3}{ }^{-}$in 18 and $\phi_{4}{ }^{+}$and $\phi_{4}{ }^{-}$in 19. Since the set of $\phi_{4}{ }^{+}$and $\phi_{4}{ }^{-}$leads to the same results

as does that of $\phi_{3}{ }^{+}$and $\phi_{3}{ }^{-}$, it is sufficient to consider only the band orbitals arising from the latter.

Bands cand d at $\Gamma$ are represented by 20 a and 20 b , respectively, and those at M by 21a and 21b, respectively. The contributions of the oxygen p orbitals of the $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges in those band orbitals are summarized in Table I. As discussed already, the



21a



21b
metal-ligand $\pi$ overlap in $9 b$ or 9 c is weaker than that in $\mathbf{9 a}, \mathbf{1 0 a}$, or 10b. In Table I, the stronger and weaker $\pi$ overlaps are

Table I. Antibonding Contributions of the Oxygen p Orbitals of Mo-O-Mo Bridges in the $\mathrm{t}_{2 \mathrm{~g}}$ Block Band Orbitals of the $\mathrm{Mo}_{2} \mathrm{O}_{9}$ Layer ${ }^{a}$

| band orbital | wave vector | unit cell orbital | intrachain |  | interchain |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | within <br> a unit cell | between nearest-neighbor unit cells |  |
| 13a | $\Gamma$ | $\phi_{1}{ }^{-}$ | N | N | N |
| 13b | $\Gamma$ | $\phi_{1}{ }^{+}$ | Y | Y | N |
| 14a | M | $\phi_{1}{ }^{-}$ | N | N | N |
| 14b | M | $\phi_{1}{ }^{+}$ | Y | Y | N |
| 16a | $\Gamma$ | $\phi_{2}{ }^{+}$ | N | N | N |
| 16b | $\Gamma$ | $\phi_{2}{ }^{-}$ | Y | Y | N |
| 17a | K' | $\phi_{2}{ }^{+}$ | Y | Y | N |
| 17b | $\mathrm{K}^{\prime}$ | $\phi_{2}{ }^{-}$ | N | N | N |
| 20a | $\Gamma$ | $\phi_{3}{ }^{-}$ | N | N | N |
| 20 b | $\Gamma$ | $\phi_{3}{ }^{+}$ | y | y | Y |
| 21a | M | $\phi_{3}{ }^{-}$ | N | N | Y |
| 21b | M | $\phi_{3}{ }^{+}$ | y | y | N |

${ }^{a}$ The presence of the antibonding contribution is indicated by the symbols Y or y , and the absence of it by the symbol N . The symbols Y and $y$ refer to the stronger and the weaker antibonding contributions discussed in connection with 9 and 10.
indicated by the symbols Y and y , respectively. 20a has no oxygen p-orbital contribution from the $\mathrm{Mo}-\mathrm{O}-\mathrm{Mo}$ bridges and thus has the same energy as 13a. 20b has oxygen p-orbital contribution from all the intra- and interchain Mo-O-Mo bridges, unlike 13b, which has oxygen p-orbital contribution only from the intrachain Mo-O-Mo bridges. Nevertheless, $\mathbf{2 0 b}$ is degenerate with $\mathbf{1 3 b}$ since the sum of two weak antibonding p-orbital contributions per unit cell is equivalent to one strong antibonding $p$-orbital contribution per unit cell. 21a has one strong antibonding p-orbital contribution per unit cell, while 21b has two weak antibonding p-orbital contributions per unit cell. Consequently, 21a and 21b are nearly degenerate, and their energies lie at the midpoint between 20a and 20b. This explains why bands c and d are dispersive along $\Gamma \rightarrow \mathrm{M}$ and why their dispersion is half as strong as that of bands a and b along $\Gamma \rightarrow \mathrm{K}^{\prime}$. In a similar way, the $\mathrm{Mo}_{2} \mathrm{O}_{\text {g }}$ layer 7 c can
also be employed to show that bands $c$ and $d$ are equally dispersive along $\Gamma \rightarrow K^{\prime}$, and their dispersion is half as strong as that of bands a and b along $\Gamma \rightarrow \mathrm{K}^{\prime}$.

## Concluding Remarks

Our tight-binding band calculations performed on a single $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer show the presence of three partially filled d-block bands. In agreement with the Zachariasen analysis, ${ }^{7}$ those bands are essentially derived from the $t_{2 g}$ levels of the $\mathrm{MoO}_{6}$ octahedra belonging to the innermost two sublayers of $\mathrm{Mo}_{6} \mathrm{O}_{17}$. Between adjacent $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layers, therefore, there is practically no overlap as far as the partially filled d-block bands are concerned. Thus our conclusions based upon a single $\mathrm{Mo}_{6} \mathrm{O}_{17}$ layer are valid in discussing the electronic properties of $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$, as supported by the excellent agreement with experiment: The Fermi surfaces of the three partially filled bands show that $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ is a 2 D metal, its CDW at 120 K results from the nesting of one of the three Fermi surfaces, and the remaining two provide electron and hole carriers below 120 K . These conclusions are also valid for $\mathrm{Na}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$ and $\mathrm{TlMO}_{6} \mathrm{O}_{17}$, which are isostructural with $\mathrm{K}_{0.9}$ $\mathrm{Mo}_{6} \mathrm{O}_{17}$. However, we note that the sizes of the hole and electron pockets of the first and third Fermi surfaces (Figures 5a,c, respectively) are not so small (about $1 / 18$ and $1 / 6$ of the Brillouin zone, respectively) as suggested by the Shubnikov-de Haas study ${ }^{2 d}$ on $\mathrm{K}_{0.9} \mathrm{Mo}_{6} \mathrm{O}_{17}$. A further experimental study is necessary. Finally, our orbital interaction analysis for the $t_{2 g}$-block bands of the innermost two sublayers of $\mathrm{Mo}_{6} \mathrm{O}_{17}$ shows that their essential dispersion characteristics are simply governed by whether or not the orbitals of bridging oxygen atoms can mix with the molybdenum $t_{2 g}$ orbitals.

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# The Variable-Temperature ESR Characterization of the Fluxional d ${ }^{9}$ Complex $\operatorname{Bis}\left(1,2-\mathrm{bis}\left(\right.\right.$ diphenylphosphino)ethane)rhodium(0), $\mathrm{Rh}(\mathrm{dppe})_{2}{ }^{0}$ 

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#### Abstract

A variable-temperature ESR study of the paramagnetic complex $\mathrm{Rh}(\mathrm{dppe})_{2}{ }^{0}$ has been performed. The complex is found to be fluxional on the ESR time scale. At 270 K the ESR spectrum of the complex in toluene consists of a symmetric five-line pattern with $\langle\boldsymbol{A}\rangle_{\mathrm{P}}$ of 52 G . Upon cooling, an alternating line width effect is seen, and the spectrum changes to a pattern showing coupling to two different pairs of $P$ donors with hyperfine couplings of 37.5 and 65 G . Analysis of the dynamic behavior leads to activation parameters $\Delta H^{*}$ and $\Delta S^{*}$ of $3.54 \mathrm{kcal} / \mathrm{mol}$ and $-4.6 \mathrm{cal} /(\mathrm{mol} \mathrm{K})$, respectively, for the exchange process. Possible mechanisms for the exchange are pairwise lengthening and shortening of the $\mathrm{Rh}-\mathrm{P}$ bonds and angular distortion of the complex to a species of $C_{2}$ symmetry. A bimolecular equilibrium is also observed by a significant decrease in the intensity of the ESR signal as the temperature is lowered. The $\Delta H$ and $\Delta S$ values for this equilibrium are $-13.4 \mathrm{kcal} / \mathrm{mol}$ and -49.8 $\mathrm{cal} /(\mathrm{mol} \mathrm{K})$, respectively, consistent with either a dimerization or the formation of an intimate ion pair, $\left[\mathrm{Rh}(\mathrm{dppe})_{2}\right]^{+}\left[\mathrm{Rh}(\mathrm{dppe})_{2}\right]^{-}$, for this sterically crowded system. The complex $\mathrm{Rh}(\mathrm{dppe})_{2}{ }^{0}$ is also found to react with $\mathrm{H}_{2}$ at room temperature to form the known hydride complex $\mathrm{RhH}(\mathrm{dppe})_{2}$.


The fluxionality of inorganic and organometallic compounds has been vigorously studied during the last two decades. ${ }^{1}$ In
general, this type of dynamic behavior in solution has been followed by variable-temperature NMR investigation from which rate


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